MATH 20D: Differential Equations Spring 2023
Homework 1

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Make sure you show all your workings.
Points will be awarded for clear explanations, not just for arriving at the correct solution.
Remember to list the sources you used when completing the assignment.

Question. (20 Points) *Select a field of study (for example, physics, geology, biology, economics, political science, ectera) and find an example of a differential equation (or a system of differential equations) which is used to analyze a problem in your choosen field. Answer the following question about your choosen differential equation(s).*

(i) What is the statement of your choosen differential equation(s)? What do the variables in the equation signify? How does this equation arise in relation to your choosen field of study?

(ii) List which variables in the equation are independent and which variables are dependent?

(iii) Is the equation a partial differential equation or an ordinary differential equation?

(iv) Is the equation a linear differential equation or a non-linear differential equation?

(v) Give a qualitative description to an interesting solution of the equation. Intepret this solution in terms of the real world applications of your equation?

Question. (5 points) *Let c_1 and c_2 be constants. Verify that the function $\phi(x) = c_1e^x + c_2e^{-2x}$ is a solution to the linear equation*

$$\frac{d^2y}{dx^2} + \frac{dy}{dx} - 2y = 0.$$

Determine c_1 and c_2 so that $\phi(0) = 2$ and $\phi'(0) = 1$.

Question. (10 points) *In each of the parts (a), (b), (c), and (d) below, determine whether the given relation is an implicit solution to the given differential equation. You may assume that the relationship does define y implicitly as a function of x.*

(a)

$$x^2 + y^2 = 4, \qquad \frac{dy}{dx} = \frac{x}{y}.$$

(b)

$$y - \log(y) = x^2 + 1, \qquad \frac{dy}{dx} = \frac{2xy}{y-1}.$$

(c)

$$x^2 - \sin(x+y) = 1, \qquad \frac{dy}{dx} = 2x\sec(x+y) - 1.$$

(d)

$$e^{xy} + y = x - 1, \qquad \frac{dy}{dx} = \frac{e^{-xy} - y}{e^{-xy} + x}.$$

Question. (20 points) *Let $\phi(x)$ denote the solution to the initial value problem*

$$\frac{dy}{dx} = x - y, \qquad y(0) = 1.$$

(a) Show that $\phi''(x) = 1 - \phi'(x) = 1 - x + \phi(x)$.
(b) Argue that the graph of ϕ is decreasing for x near zero and that as x increases from zero, $\phi(x)$ decreases until it crosses the line $y = x$.
(c) Write (x^*, y^*) for the coordinates of the first point to the right of the y-axis where the curve $y = \phi(x)$ intersects the line $y = x$. Show that $\phi'(x^*) = 0$.
(d) Argue that the curve $y = \phi(x)$ has a local minimina at $x = x^*$.
(e) (Optional) Sketch the direction field of the differential equation $dy/dx = x - y$ using the method of isoclines.
(f) (Optional) Sketch the solution $y = \phi(x)$ using the direction field of part (e).
(g) (Optional) Explain why the curve $y = \phi(x)$ cannot intersect the curve $y = x - 1$.